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Abstract

We correct Discussion 5.2. in our prior work: doi: 10.1016/j.neucom.2017.04.068.

1 Error

1.1 Exact Value of Real Log Canonical Threshold of Reduced Rank Regression

$$\lambda_{RRR}^{r} = \begin{cases} \left\{2(H+r)(M+N)-(M-N)^2-(H+r)^2\right\}/8 \\ \quad \text{if}(N+r<M+H \wedge M+r<N+H \wedge H+r<M+N \wedge M+H+N+r:\text{even}) \\ \left\{2(H+r)(M+N)-(M-N)^2-(H+r)^2+1\right\}/8 \\ \quad \text{if}(N+r<M+H \wedge M+r<N+H \wedge H+r<M+N \wedge M+H+N+r:\text{odd}) \\ (HM-Hr+Nr)/2 \quad \text{if}(M+H<N+r) \\ (HN-Hr+Mr)/2 \quad \text{if}(N+H<M+r) \\ MN/2 \quad \text{if}(M+N<H+r). \end{cases} \tag{1}$$

1.2 Proof of Inequality

(Case 1) $r=0$ i.e. in the case of Lemma 3.1.

(Case 1-1) $N<M+H \wedge M<N+H \wedge H<M+N \wedge M+H+N : \textit{even}$.

We assume $N \leqq M$, i.e. , $\lambda_{NMF}^{3.1.} = HM/2$. Owing to equality (2),

$$\begin{aligned} \lambda_{RRR}^{0} &= \left\{2H(M+N)-(M-N)^2-H^2\right\}/8 \\ &= \left\{2(MH+HN+NM)-M^2-N^2-H^2\right\}/8. \\ \lambda_{RRR}^{0}-\lambda_{NMF}^{3.1.} &= -\left\{M^2+H^2+N^2-2(MH+HN+NM)-4HM\right\}/8 \\ &= -\left\{M^2+H^2+N^2-2(-MH+HN+NM)\right\}/8 \\ &= -\left[(-M)^2+(-H)^2+N^2+2\{(-M)(-H)+(-H)N+N(-M)\}\right]/8 \\ &= -(N-M-H)^2/8<0 \quad (N<M+H). \end{aligned}$$

Therefore

$$\lambda^0_{RRR} < \lambda^{3.1.}_{NMF}.$$

If $N > M$, that can be derived in the same way as above.

(Case 1-2) $N < M + H \wedge M < N + H \wedge H < M + N \wedge M + H + N : odd$.

We assume $N \leqq M$, i.e. , $\lambda^{3.1.}_{NMF} = HM/2$. In the same way as Case 1-1,

$$\begin{aligned} \lambda^0_{RRR} - \lambda^{3.1.}_{NMF} &= 1/8 - (N - M - H)^2/8 \\ &= 1^2/8 - (M + H - N)^2/8 \\ &= -(M + H - N + 1)(M + H - N - 1)/8. \end{aligned}$$

We derive $N+1 \leqq M+H$ by reductio ad absurdum. We suppose $M+H < N+1$. Using the assumption of Case 1-2,

$$N < M + H < N + 1$$

$M + H$ and N are natural numbers, thus the above inequality is inconsistent. That is why $N + 1 \leqq M + H$ and

$$-(M + H - N + 1)(M + H - N - 1) \leqq 0.$$

Therefore, $\lambda^0_{RRR} \leqq \lambda^{3.1.}_{NMF}$. If $N > M$, that can be derived in the same way as above.

2 Correct

2.1 Exact Value of Real Log Canonical Threshold of Reduced Rank Regression

$$\lambda^r_{RRR} = \begin{cases} \left\{2(H + r)(M + N) - (M - N)^2 - (H + r)^2\right\}/8 \\ \quad \text{if}(N + r \leqq M + H \wedge M + r \leqq N + H \wedge H + r \leqq M + N \wedge M + H + N + r : \text{even}) \\ \left\{2(H + r)(M + N) - (M - N)^2 - (H + r)^2 + 1\right\}/8 \\ \quad \text{if}(N + r \leqq M + H \wedge M + r \leqq N + H \wedge H + r \leqq M + N \wedge M + H + N + r : \text{odd}) \\ (HM - Hr + Nr)/2 \quad \text{if}(M + H < N + r) \\ (HN - Hr + Mr)/2 \quad \text{if}(N + H < M + r) \\ MN/2 \quad \text{if}(M + N < H + r). \end{cases} \tag{2}$$

2.2 Proof of Inequality

(Case 1) $r = 0$ i.e. in the case of Lemma 3.1..

(Case 1-1) $N \leqq M + H \wedge M \leqq N + H \wedge H \leqq M + N \wedge M + H + N : even$.

We assume $N \leqq M$, i.e. , $\lambda_{NMF}^{3.1.} = HM/2$. Owing to equality (2),

$$\begin{aligned}
\lambda_{RRR}^{0} &= \left\{2H(M+N) - (M-N)^2 - H^2\right\}/8 \\
&= \left\{2(MH+HN+NM) - M^2 - N^2 - H^2\right\}/8. \\
\lambda_{RRR}^{0} - \lambda_{NMF}^{3.1.} &= -\left\{M^2 + H^2 + N^2 - 2(MH+HN+NM) - 4HM\right\}/8 \\
&= -\left\{M^2 + H^2 + N^2 - 2(-MH+HN+NM)\right\}/8 \\
&= -\left[(-M)^2 + (-H)^2 + N^2 + 2\{(-M)(-H) + (-H)N + N(-M)\}\right]/8 \\
&= -(N-M-H)^2/8 \leqq 0
\end{aligned}$$

Therefore

$$\lambda_{RRR}^{0} \leqq \lambda_{NMF}^{3.1.}.$$

If $N > M$, that can be derived in the same way as above.

(Case 1-2) $N \leqq M+H \wedge M \leqq N+H \wedge H \leqq M+N \wedge M+H+N$: *odd*.

We assume $N \leqq M$, i.e. , $\lambda_{NMF}^{3.1.} = HM/2$. In the same way as Case 1-1,

$$\begin{aligned}
\lambda_{RRR}^{0} - \lambda_{NMF}^{3.1.} &= 1/8 - (N-M-H)^2/8 \\
&= 1^2/8 - (M+H-N)^2/8 \\
&= -(M+H-N+1)(M+H-N-1)/8.
\end{aligned}$$

If $H = 0$, $N+1 \leqq M$ is attained because of that $M+N$ must be odd and $N \leqq M$, hence $\lambda_{RRR}^{0} \leqq \lambda_{NMF}^{3.1.}$. Else, owing to $H \geqq 1$ and $N \leqq M$,

$$\begin{aligned}
N+1 &\leqq M+1 \\
&\leqq M+H.
\end{aligned}$$

We also get $N-1 \leqq N+1 \leqq M+H$ thus

$$-(M+H-N+1)(M+H-N-1) \leqq 0.$$

Therefore,

$$\lambda_{RRR}^{0} \leqq \lambda_{NMF}^{3.1.}.$$

If $N > M$, that can be derived in the same way as above.

References

[1] N. Hayashi and S. Watanabe, "Upper bound of Bayesian generalization error in non-negative matrix factorization," *Neurocomputing*, 2017, to appear.